MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

BSA1024 – STATISTICS
(All sections / Groups)

02 MARCH 2020
9.00a.m 11.00a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **THIRTEEN (13)** printed pages with:
   Section A: **Ten (10)** multiple choice questions (20%)
   Section B: **Three (3)** structured questions (80%)

2. Answer **all** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Formula and Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restrictions.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple-choice answer sheet.

1. __________ is used to compare the variation or dispersion in two or more sets of data even though they are measured in different units.

   A. Range
   B. Standard deviation
   C. Coefficient of variation
   D. Interquartile range

2. The percent of total variation of the dependent variable Y is explained by the set of independent X is measured by:

   A. Coefficient of correlation
   B. Coefficient of determination
   C. Standard error of estimate
   D. Variation for dependent variable Y

3. If a parameter is computed from a set of data, which of the following statements is TRUE?

   A. The data are obtained from a census.
   B. The data are obtained from a random sample.
   C. The data is from a sample that is representative of the population.
   D. The data must be quantitative.

4. The probability function of a random variable, X is defined as below:

X	-1	-2	0	1	2
P(X)	m	$2m$	$3m$	$4m$	$5m$

   What is the value of m?

   A. 0.25
   B. 0.125
   C. 1/15
   D. 1

5. In binomial experiment, the successive trials are __________.

   A. Dependent
   B. Independent
   C. Mutually exclusive
   D. Fixed

Continued...

6. Which one of the following variables is *not* quantitative?

   A. Age of a person
   B. Gender of a person
   C. Number of dreams recalled
   D. Number of students in BMS1024 class

7. During the grand opening week of a bicycle shop, it has offers a wheel of discount saving. After customers select the items they wish to purchase, they spin the wheel to determine the discount they will receive. The wheel is divided into 12 slices. 6 slices are red and award a 10% discount, 3 slices are white and award a 20% discount, and 2 slices are blue and award a 40% discount. The remaining slice is gold and awards a 100% discount. The probability that a customer gets at least 40% discount is

   A. 0.25
   B. 0.1667
   C. 0.0625
   D. 0.75

8. A major department store chain is interested in estimating the mean amount its credit card customers spent on their first visit to the chain's new store in the mall. Fifteen credit card accounts were randomly sampled and analyzed with the following results: $\bar{x} = RM50.50$ and $s = RM20$. Assuming the distribution of the amount spent on their first visit is normal, what is the shape of the sampling distribution of the sample mean that will be used to create the desired confidence interval for μ?

   A. approximately normal with a mean of $RM50.50$
   B. a standard normal distribution
   C. a *t*-distribution with 15 degrees of freedom
   D. a *t*-distribution with 14 degrees of freedom

9. A computer software developer would like to use the number of downloads (in thousands) for the trial version of his new software to predict the amount of revenue (in thousands of dollars) on selling the full version of his new software. A table is provided as below to display an output from a simple linear regression analysis:

Multiple R	0.8691
R Square	0.7554
Adjusted R Square	0.7467
Standard Error	44.4765
Observations	30

Continued...

Which of the following is the correct interpretation for the coefficient of determination?

A. 75.54% of the variation in the number of downloads can be explained by the variation in revenue.
B. 74.67% of the variation in the number of downloads can be explained by the variation in revenue.
C. 86.91% of the variation in revenue can be explained by the variation in the number of downloads.
D. 75.54% of the variation in revenue can be explained by the variation in the number of downloads.

10. In 2001, moving companies are required by the government to publish a Carrier Performance Report. In additional, they need to include the number of shipments which claimed *RM*50 or greater for damage was filed. There were two companies named; *AK-Move* and *Big-Move*, decided to estimate this figure by sampling their records, and they were reported their data in the following table:

Name of company	***AK-Move***	***Big-Move***
Total of shipments sampled	900	750
Number of shipments with a damage claimed $\geq$ *RM*50	162	60

Which of the hypothesis statistical test would be used to analyze this data?

A. *t-test* for the difference between two means.
B. *F-test* for the ratio of variances.
C. *Z-test* for the difference between two proportions.
D. Separate variance binomial test for the difference between two means.

Continued...

SECTION B: STRUCTURED QUESTIONS (80) MARKS)

There are THREE (3) questions in this section. Candidates MUST answer ALL THREE questions.

Question 1 (30 Marks)

a) Isabel is going to play one badminton match and one tennis match. The badminton match has been scheduled one week earlier than tennis match. Based on the prediction by her coach, the chance that she will win the badminton match is $\frac{9}{10}$. Other than that, the coach believed that if Isabel wins the badminton match, the chance that she will win the tennis match is $\frac{3}{5}$. But if she loses the badminton match, she has $\frac{2}{3}$ chance of losing the tennis match too.

(i) Draw a tree diagram and the probabilities for each events involved for the above scenario.

[8 marks]

(ii) Find the probability that Isabel will win for the tennis match.

[4 marks]

(iii) The matches have been re-scheduled. Given that Isabel won the tennis match, what is the probability that she will also win the badminton match?

[3 marks]

b) According to the 2018 population survey conducted by Statistics Department of Malaysia, 32% of the Malaysia population with age 25 years old or over has completed a bachelor' degree or more. Given a random sample of 10 people with age 25 years old or over,

(i) What is the probability that at most 2 of the selected people has completed a bachelors degree or more?

[4 marks]

(ii) What is the average number of people who have completed a bachelors degree or more from the sample?

[2 marks]

c) A test has been devised to measure student's level of motivation during high school. The higher the score the greater the motivation to do well at school. The motivation scores on the test are approximately normally distributed with a mean of 25 and a standard deviation of 6.

Continued...

(i) What is the probability that a student taking this test will obtain scores between 20 to 28?

[5 marks]

(ii) Eliya has been told that 35% of the students taking the test have higher motivation scores than she does. What is Eliya's score?

[4 marks]

Question 2 (20 Marks)

a) In Europe countries, the demand on purchasing homes located only a few steps away from beach is increasing among rich people there. Based on a realtor's claim, the oceanfront homes (directly on the beach) have greater value than oceanside homes, which are not directly on the beach. Listed below are fair market values (in thousands of dollars) of randomly 10 selected homes on Long Beach Island in New Jersey.

Oceanfront	2199	3750	1725	2398	2799	4521	1865	2553	3025	1755
Oceanside	700	1355	795	1575	759	1865	2045	956	825	1348

At 5 percent significance level, can we conclude that oceanfront homes have greater market value compared to oceanside homes?

[10 marks]

b) The health insurance premium is much higher if an insurer is a smoker. It is due to smokers are more likely to claim insurance due to suffering an early death or a critical illness. An insurance company believed that the male adult is more likely to be a smoker than female adult. From a telephone poll, the result was recorded as below:

	Male Adult	***Female Adult***
Number of respondent	605	195
Number of smoker	351	41

At the 5 percent significance level, is there enough evidence to support the company's belief?

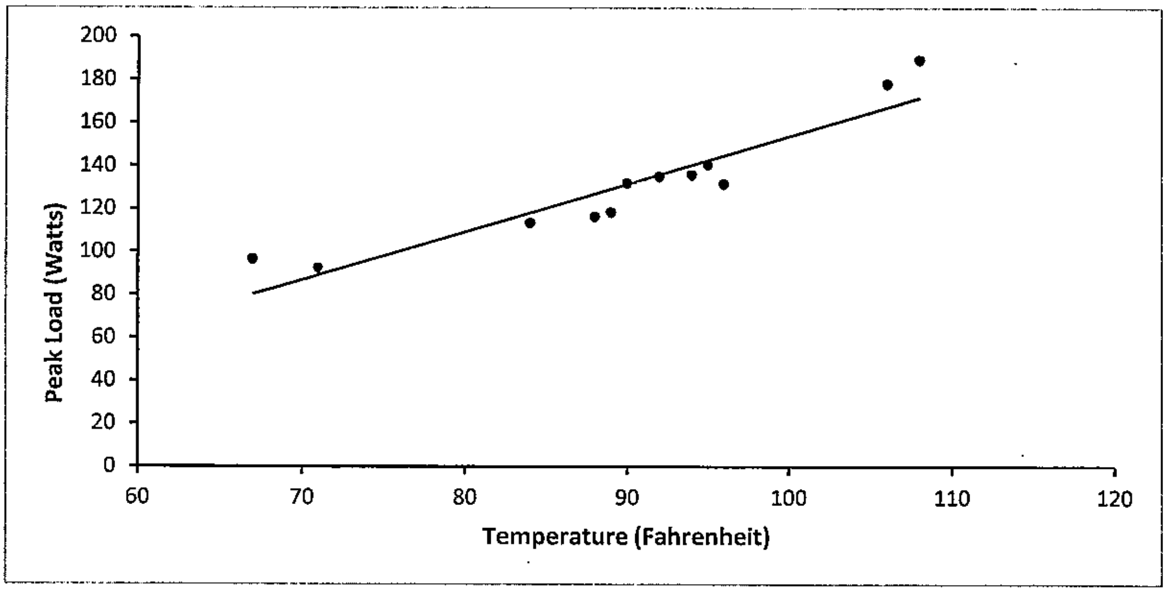
[10 marks]

Question 3 (30 Marks)

a) Peak power load is the maximum amount of power which need to be generate daily to meet customer's demand. A power company wants to use daily high temperature to predict daily peak power load during the summer season when demand is greatest. These data, scatterplot and summary output of the relationship between the two variables are shown below:

Continued...

Temperature (Fahrenheit)	*Peak Load (Watts)*
94	136
96	131.7
95	140.7
108	189.3
67	96.5
88	116.4
89	118.5
84	113.4
90	132
106	178.2
71	92.5
92	135.1



Summary Output

Regression Statistics	
Multiple R	0.9342
R Square	0.8728
Adjusted R Square	0.8600
Standard Error	10.7661
Observations	12

Continued...

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	7950.967864	7950.968	68.59715	8.67897E-06
Residual	10	1159.081302	115.9081		
Total	11	9110.049167			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-69.4402	24.4825	-2.83632	0.017659
Temperature	2.2348	0.2698	8.28234	8.68E-06

(i) State the dependent variable and the independent variable for the above regression model.

[2 marks]

(ii) Write the least square regression line for the above relationship between the two variables. State the unit of measurement for each variable.

[4 marks]

(iii) What do the coefficient of the regression line tell you about the relationship between peak power load and the temperature during summer?

[3 marks]

(iv) Determine the coefficient of correlation and discuss the role of this coefficient value for this model.

[3 marks]

(v) Predict the peak power load if the temperature is set at 110 Fahrenheit. Is this estimation reliable? Explain.

[4 marks]

(vi) State the coefficient of determination and describe what it tells you.

[4 marks]

b) A table below is consisting selected consumer goods for 2009 and 2019:

Product	2009		2019	
	Price (RM)	Quantity	Price (RM)	Quantity
Food	340	115	345	90
Rent	600	16	600	13
Drinks	120	90	150	120
Transportation	250	43	260	56

Compute and interpret the Laspeyres Price Index (LPI) and Paashe Price Index (PPI) for 2019 using 2009 as the base period.

[10 marks]

End of Page.

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\overline{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\overline{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution $B(n, p)$ where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) =

$$\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) =

$$\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{p^\wedge}$

Where $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \frac{(Z_{\alpha/2})^2\,\hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
One Sample Proportion Test	
$z = \frac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$	
Two Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$
Two Sample Proportion Test	
$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $p = \frac{X_1+X_2}{n_1+n_2}$	
where X_1 and X_2 are the number of successes from each population	

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left((\sum X)^2 / n\right)\right]\left[\sum Y^2 - \left((\sum Y)^2 / n\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	*1*	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

Where p= number of predictor

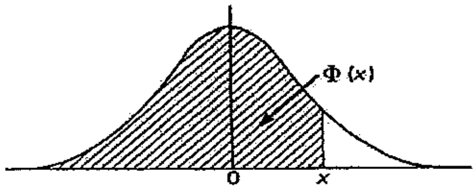
G. INDEX NUMBERS

Simple Price Index $P = \frac{p_t}{p_0} \times 100$	**Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index $P = \frac{\sum p_t}{\sum p_0}(100)$	**Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	**Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ Price\ Index)(Paasche\ Price\ Index)}$
Paasche Price Index $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	**Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

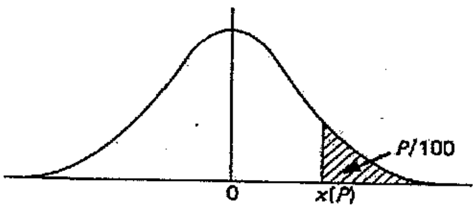
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



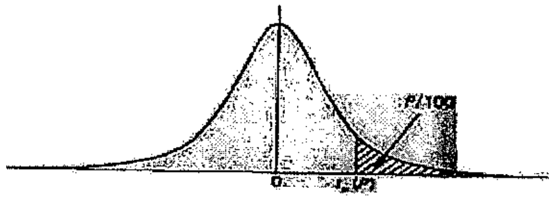
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}}\frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)}\int_{t_\nu(P)}^{\infty}\frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
ν = 1	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	14	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	13	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	11	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

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